Formula Sheet

$x(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos(2\pi f_0 nt) + \sum_{n=1}^{\infty} b_n \sin(2\pi f_0 nt)$	$X(\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt$
$a_0 = \frac{1}{T_0} \int_{T_0} x(t) dt$	$x(t) = \mathcal{F}^{-1}[X(\omega)] = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\omega) e^{j\omega t} d\omega$
$a_n = \frac{2}{T_0} \int_{T_0} x(t) \cos(2\pi n f_0 t) dt$	$c_1 x_1(t) + c_2 x_2(t) \leftrightarrow c_1 X_1(\omega) + c_2 X_2(\omega)$
$b_n = \frac{2}{T_0} \int_{T_0} x(t) \sin(2\pi n f_0 t) dt$	$x(t-c) \leftrightarrow X(\omega) e^{-j\omega c}$
$x(t) = \underbrace{C_0}_{\text{dc component}} + \sum_{n=1}^{\infty} \underbrace{C_n \cos(2\pi n f_0 t + \theta_n)}_{\text{nth harmonic}}$	$x(t) e^{j\omega_0 t} \leftrightarrow X(\omega - \omega_0)$
$C_0 = a_0$	$x(t)\cos(\omega_0 t) \leftrightarrow \frac{1}{2}[X(\omega + \omega_0) + X(\omega - \omega_0)]$
$C_n = \sqrt{a_n^2 + b_n^2}$	$\frac{d}{dt} x(t) \rightleftarrows j\omega X(\omega)$
$\theta_n = -\tan^{-1}\left(\frac{b_n}{a_n}\right)$	$\int_{-\infty}^{t} x(\tau) d\tau \rightleftarrows \frac{1}{j\omega} X(\omega)$
$x(t) = \sum_{n=-\infty}^{\infty} F_n e^{j 2\pi f_0 nt}$	$X(t) \rightleftarrows 2\pi x(-\omega)$
$F_n = \frac{1}{T_o} \int_{T_o} x(t) e^{-j 2\pi f_0 nt} dt$	$h(t) = S\left\{\delta(t)\right\}$
$F_{-n} = F_n^* \longrightarrow \lvert F_n \rvert = \lvert F_{-n} \rvert \longrightarrow \angle F_n = -\angle F_{-n}$	$y(t) = S\{x(t)\} = x(t) \star h(t)$
$F_n = 0.5(a_n - jb_n)$	$y(t) = \int_{-\infty}^{\infty} x(\tau) h(t-\tau) d\tau$
$F_{-n} = F^*_n = 0.5(a_n + jb_n)$	$x(at) \rightleftarrows \frac{1}{\lvert a \rvert} X\left(\frac{\omega}{a}\right)$
$F_n = 0.5 C_n \angle \theta_n = 0.5 C_n e^{j\theta_n}$	
$C_n = 2\lvert F_n \rvert$	
$\theta_n = \angle F_n$	
$C_0 = a_0 = F_0$	

A Short Table of Fourier Transforms

	$f(t)$	$F(\omega)$	
1	$e^{-at}u(t)$	$\dfrac{1}{a+j\omega}$	$a>0$
2	$e^{at}u(-t)$	$\dfrac{1}{a-j\omega}$	$a>0$
3	$e^{-a\|t\|}$	$\dfrac{2a}{a^2+\omega^2}$	$a>0$
4	$te^{-at}u(t)$	$\dfrac{1}{(a+j\omega)^2}$	$a>0$
5	$t^n e^{-at}u(t)$	$\dfrac{n!}{(a+j\omega)^{n+1}}$	$a>0$
6	$\delta(t)$	1	
7	1	$2\pi\delta(\omega)$	
8	$e^{j\omega_0 t}$	$2\pi\delta(\omega-\omega_0)$	
9	$\cos\omega_0 t$	$\pi[\delta(\omega-\omega_0)+\delta(\omega+\omega_0)]$	
10	$\sin\omega_0 t$	$j\pi[\delta(\omega+\omega_0)-\delta(\omega-\omega_0)]$	
11	$u(t)$	$\pi\delta(\omega)+\frac{1}{j\omega}$	
12	$\operatorname{sgn} t$	$\frac{2}{j\omega}$	
13	$\cos\omega_0 t\, u(t)$	$\frac{\pi}{2}[\delta(\omega-\omega_0)+\delta(\omega+\omega_0)]+\frac{j\omega}{\omega_0^2-\omega^2}$	
14	$\sin\omega_0 t\, u(t)$	$\frac{\pi}{2j}[\delta(\omega-\omega_0)-\delta(\omega+\omega_0)]+\frac{\omega_0}{\omega_0^2-\omega^2}$	
15	$e^{-at}\sin\omega_0 t\, u(t)$	$\frac{\omega_0}{(a+j\omega)^2+\omega_0^2}$	$a>0$
16	$e^{-at}\cos\omega_0 t\, u(t)$	$\frac{a+j\omega}{(a+j\omega)^2+\omega_0^2}$	$a>0$
17	$\operatorname{rect}\left(\frac{t}{\tau}\right)$	$\tau\operatorname{sinc}\left(\frac{\omega\tau}{2}\right)$	
18	$\frac{W}{\pi}\operatorname{sinc}(Wt)$	$\operatorname{rect}\left(\frac{\omega}{2W}\right)$	
19	$\Delta\left(\frac{t}{\tau}\right)$	$\frac{\tau}{2}\operatorname{sinc}^2\left(\frac{\omega\tau}{4}\right)$	
20	$\frac{W}{2\pi}\operatorname{sinc}^2\left(\frac{Wt}{2}\right)$	$\Delta\left(\frac{\omega}{2W}\right)$	
21	$\displaystyle\sum_{n=-\infty}^{\infty}\delta(t-nT)$	$\displaystyle\omega_0\sum_{n=-\infty}^{\infty}\delta(\omega-n\omega_0)$	$\omega_0=\frac{2\pi}{T}$
22	$e^{-t^2/2\sigma^2}$	$\sigma\sqrt{2\pi}e^{-\sigma^2\omega^2/2}$	